Josephson coupling of Bose-Einstein condensates of exciton-polaritons in semiconductor microcavities

D. Read,[1] Yuri G. Rubo,[2] and A. V. Kavokin[1,3]

[1]*School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom*

[2]*Centro de Investigación en Energía, Universidad Nacional Autónoma de México, Temixco, Morelos 62580, Mexico*

[3]*Dipartimento di Fisica, University of Rome II, 1, via della Ricerca Scientifica, Rome 00133, Italy*

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We show theoretically that weak (Josephson) coupling between two localized Bose-Einstein condensates of exciton-polaritons may induce the vector polarization locking in the system. Polarization correlations between two condensates appear if the lifetime of polaritons is long enough to allow for the efficient spin relaxation. The correlations are suppressed by local effective magnetic fields originated by strain or photonic disorder.

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I. INTRODUCTION

A microcavity polariton is a superposition of a confined photon and a Wannier-Mott exciton in a Fabry-Perot microcavity containing a semiconductor medium.[1] The composite nature of polaritons results in a small effective mass and short lifetime due to the photonic component while the excitonic component results in effective polariton interactions/scatterings with other excitations of the medium (e.g., other polaritons, phonons). These factors combine to allow for the Bose-Einstein condensation (BEC) of polaritons at extremely high critical temperatures relative to other condensate forming systems.

Recent experimental work has led to the realization of polariton BEC at room[2] and cryogenic temperatures.[3,4] BEC can be characterized by its complex order parameter, $\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\phi(\mathbf{r})}$, associated with the macroscopic condensed wave function and defined in terms of the condensate density ρ and a well-defined phase ϕ. BEC is a symmetry-breaking phase transition with ϕ being chosen spontaneously by the system. In this work we use the kinetic theory of spontaneous formation of the order parameter.[5] We demonstrate that the buildup of a stochastic polarization above threshold is directly proportional to a buildup in the condensate order parameter. This allows for an experimentally simple measurement of the spontaneous symmetry breaking required for BEC through polarization measurements. The polarization buildup is accompanied by appearance of spatial correlations, $g^{(1)}(\mathbf{r},\mathbf{r}')$, proposed as the criterion for BEC phase transition.[6] We argue that the measurement of a spontaneous stochastic polarization may be a more stringent criterion in realistic microcavities. Indeed, spatial coherence can be assumed in systems such as micropillars[7] and is likely a feature of localized condensates in planar microcavities once the lasing threshold is surpassed,[2] making it an unsuitable criterion for defining the phase transition. The spontaneous polarization build up defines BEC in localized condensates where the spatial coherence can be formed prior to the order parameter formation.

In recent experiments, it has been shown in the near-field emission profile that multiple condensates are formed within the microcavity sample.[8] A buildup of long-range spatial coherence and uniformly oriented emission polarization across the spatially separate condensates is seen above threshold. This implies a coupling between the condensates. We propose that the dc Josephson effect,[9,10] when applied as an extension to the kinetic model, can successfully account for this coupling and explain the polarization locking and spatial coherence buildup in the system. Furthermore, through the symmetry-breaking effect of differing local fields for each condensate (e.g., produced by local strains at different regions of the sample) the polarization locking can be affected through field orientations.

The Josephson effect was predicted and observed for two superconductors separated by a thin insulating layer. It was later discovered for superfluid helium[11] and interacting atomic condensates.[12] Theoretical work has recently been published describing the polarization dynamics of a pair of fully correlated polariton condensates[13] and the phase correlation of Josephson-coupled condensates as a function of detuning.[14] The difference in respect of both these works is that we take into account the stochastic nature of BEC formation and examine the buildup of polarization correlations. We assume the condensates are not detuned from each other and therefore are in the steady-state regime as described in Ref. 14.

II. FORMALISM

We consider BEC into two spin degenerate spatially separated ground-state levels from one incoherent reservoir of polaritons. In general, the semiclassical Boltzmann equation needs to be solved to determine the population of noncondensed states, however, we adopt a simple model where all noncondensed polaritons, $N_r(t)$, are treated as equivalent and belonging to a single reservoir with a population time dependence

$$\frac{dN_r}{dt} = -\Gamma_r N_r - W(t)[n(t)+1] + P(t). \tag{1}$$

Here Γ_r is the polariton decay rate in the reservoir, $W(t)$ is the income rate for which we have assumed the scattering mechanism is predominantly polariton phonon and therefore takes the form $W(t)=rN_r(t)$, where r is the condensation rate, $n(t)$ is the full instantaneous populations of two condensates,

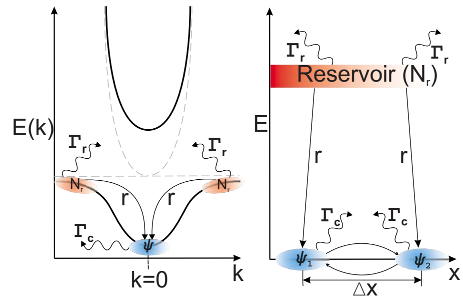

FIG. 1. (Color online) Schematic of polariton system showing the polariton dispersion in k space (left) and the real-space layout of two Josephson-coupled condensates with a separation of Δx (right). The scale for E is not the same for both diagrams. Both ground-state condensates are fed by a common reservoir with $N_r(t)$ polaritons on the excitonlike part of the dispersion and are Josephson coupled with coherent hopping parameter J.

and $P(t)$ is the pump. We model a pulsed pump and assume a pulse length much shorter than the condensate lifetime allowing us to take $P(t)$ as an initial condition. We assume the reservoir is completely depolarized, due to rapid spin-relaxation processes.

The dynamics of the two-condensate order parameter, $\psi_{i\sigma}(t)$, where $\sigma=\pm$ denotes the two spin (circular-polarization) components of the order parameter and $i=1,2$ labels the two condensates, is described by the Langevin-type equation,[5]

$$\frac{d\psi_{i\sigma}}{dt}=\frac{1}{2}[W(t)-\Gamma_c]\psi_{i\sigma}+\theta_{i\sigma}(t)-\frac{i}{\hbar}\frac{\delta H}{\delta\psi^*_{i\sigma}}-\gamma R_{i\sigma}, \quad (2)$$

where Γ_c^{-1} is the polariton lifetime in the condensate, $\theta_{i\sigma}(t)$ is the noise defined by correlators[15]

$$\langle\theta_{i\sigma}(t)\theta_{i'\sigma'}(t')\rangle=0, \quad (3)$$

$$\langle\theta_{i\sigma}(t)\theta^*_{i'\sigma'}(t')\rangle=(1/4)W(t)\delta_{ii'}\delta_{\sigma\sigma'}\delta(t-t'). \quad (4)$$

The third term on the RHS of Eq. (2) describes the Hamiltonian dynamical effects due to the spin splitting, the Josephson coupling between two condensates, and the spin-dependent polariton-polariton interactions,

$$H=H_s+J\sum_{\sigma}(\psi^*_{1\sigma}\psi_{2\sigma}+\psi^*_{2\sigma}\psi_{1\sigma})+\frac{1}{2}\sum_{i\sigma}[\alpha_1|\psi_{i\sigma}|^4+\alpha_2|\psi_{i\sigma}|^2|\psi_{i\bar{\sigma}}|^2], \quad (5)$$

where the spin-splitting part H_s will be defined later, J is the strength of coherent hopping between the condensates (sensitive to the condensate separation and the confining potential), and α_1 and α_2 are the on-site interaction constants of polaritons with parallel and antiparallel spins, respectively. The final term accounts for the spin relaxation of the condensate. We take a Landau-Khalatnikov[16] approach and therefore $R_{i\sigma}=\delta H/\delta\psi^*_{i\sigma}$. A schematic of this coupling can be seen in Fig. 1. A more detailed background is given in Ref. 5.

We solve the system of Langevin equations coupled to the Boltzmann reservoir numerically using a fifth-order Adams-Bashforth-Moulton predictor corrector method. We first present the results for the situation where there are no effective magnetic fields, i.e., $H_s=0$.

III. POLARIZATION CORRELATION AND PHASE COHERENCE

From polarization-resolved photoluminescence experiments the condensate Stokes parameters, or the components of the pseudospin vector $\mathbf{S}$, are directly accessible. The pseudospins can be written using the Pauli matrices $\boldsymbol{\sigma}$ as

$$\mathbf{S}_i=(1/2)(\psi_i^\dagger\cdot\boldsymbol{\sigma}\cdot\psi_i), \quad (6)$$

where we define spinors $\psi_i=(\psi_{i+},\psi_{i-})^{\mathrm{T}}$. In the calculations, the time averaging will be taken over numerous realizations of noise, and this is denoted by angular brackets as in Eq. (3). The numerical parameters used in calculations are $\Gamma_c=0.5\ \mathrm{ps}^{-1}$, $\Gamma_r/\Gamma_c=0.01$, $\alpha_1=1.8\ \mu\mathrm{eV}$, $\alpha_2/\alpha_1=-0.1$, $\gamma=1\ \mathrm{meV}^{-1}\ \mathrm{ps}^{-1}$, $J=10\hbar\Gamma_r$, and $r=10^{-4}\ \mathrm{ps}^{-1}$. The value of J varies in large limits as the distance between two condensates changes. It ranges from 0 (corresponding to the infinitely remote condensates) to about 1 meV, which is the characteristic localization energy of a condensate of exciton-polaritons. We have taken $J=0.03$ meV, which is two orders of magnitude less than the localization energy. The corresponding tunneling time between two condensates is about 20 ps, which is an order of magnitude longer than the exciton-polariton lifetime. This is a characteristic value for the weak-coupling regime of two condensates corresponding to a spatial separation on the order of several micrometer.

The most convenient way to evidence locking of random polarizations generated by the two localized condensates is by direct measurement of the two-emitter polarization correlator

$$S_{corr}(t)=\sqrt{\left|\left\langle\frac{(\mathbf{S}_1(t)\mathbf{S}_2(t))}{|S_1(t)||S_2(t)|}\right\rangle\right|}, \quad (7)$$

where the denominator represents normalization by the condensate populations. $S_{corr}(t)$ changes between 0 for uncorrelated polarizations and 1 for full correlation. It can be calculated for different values of P/P_{th}, where P_{th} is the threshold pump determined by income and outcome rates of polaritons, i.e., $\int P_{th}dt=\Gamma_c/r$. The results of these calculations are shown in Fig. 2 with the time resolved, ensemble averaged, population of the condensates (the population of each condensate is virtually identical as relaxation from the reservoir is equally probable).

During the condensate formation, the polarizations and phases $\phi_{i\sigma}=\arg(\psi_{i\sigma})$ of the condensates are stochastically chosen. The relationship between the two phases is an important quantity as correlations would evidence the buildup of spatial coherence. To explore this relationship we look at the phase difference between the components of the order parameter, $\phi_{2\sigma}-\phi_{1\sigma}$. The probability density for this quantity is shown in Fig. 3 at three different times above threshold pumping.

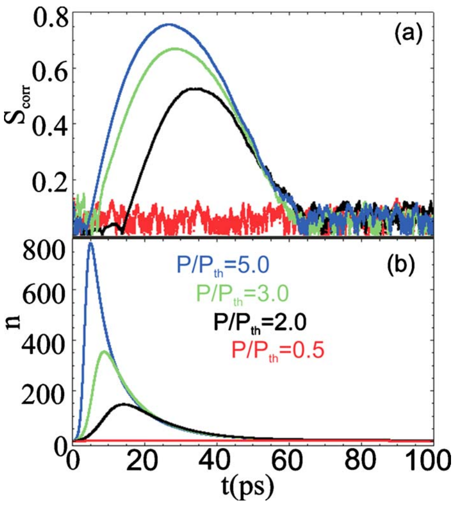

FIG. 2. (Color online) Below and above threshold time dynamics of the (a) polarization correlator defined by Eq. (7) and (b) condensate occupation. The polarization correlations are seen as long as the condensates are macroscopically occupied. The maximum degree of correlation is dependent on the condensate occupation.

When the condensate occupation is less than 1 the probability density as shown in Fig. 3 is that of the difference between two uniform random variables spanning the interval $-\pi$ to π. During the period of significant population [as can be seen in Fig. 2(b)] the condensates have the greatest probability of assuming phase differences of π or $-\pi$, which are equivalent. We believe this phase difference is due to the rotation of the order parameter from the Josephson coupling. As we are dealing with a stochastic process either condensate 1 or 2 will breach threshold occupation first and as the Josephson coupling enters our system as an imaginary term this acts to shift the phase of the later forming condensate. This shift is by a factor $\pi/2$. Once the later forming condensate becomes macroscopically occupied it acts in the same way on the initially formed condensate and we see a total phase difference of π or $-\pi$. Although not shown in Fig. 3, once the condensate occupation has decayed below 1 (for $t>60$ ps) the distribution reverts to the same form as at $t=1$ ps. A figure identical to Fig. 3 is obtained for the minus circularly polarized components of the condensates.

Below threshold no correlations are seen between the condensates, also the total polarization degree of the condensates, which is directly proportional to their order parameters, is close to zero. There is no macroscopic population of the condensate as can be seen in Fig. 2 and therefore no Josephson current between them.

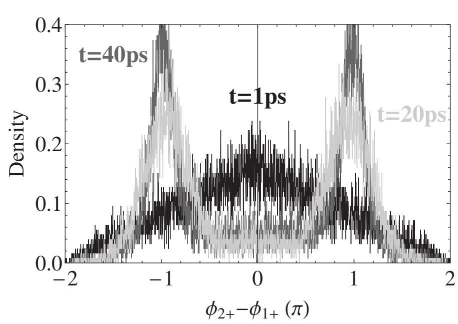

FIG. 3. Probability density of phase difference between the two condensates at times before and during macroscopic occupation. For all times $P/P_{th}=3.0$.

Above threshold the condensates populate macroscopically with a narrowing of the emission peak with increasing pump power. The polarization degree approaches 1 (without polariton-polariton interactions it reaches 1 for all values of pump, however, when they are included a gradual decrease is seen above threshold[5]) indicating a clear spontaneous symmetry breaking, the "smoking gun" for BEC. The polarization correlator increases with increasing pump to a maximum value of $S_{corr}(t)\approx 0.75$. This evidences the polarization locking of the two condensates due to the dc Josephson effect. Importantly, the lifetime of the correlation is roughly constant regardless of pump power, in opposition to the condensate occupations. This indicates that any macroscopic population of the condensates is enough for a significant polarization locking effect between them. The spatial coherence builds up in a similar way, as evidenced by the results in Fig. 3. This shows that the spatial coherence and spontaneous polarization both carry information on the order parameter of BEC. In systems of large size the buildup of spatial coherence and of spontaneous polarization equally characterizes the BEC threshold. However, in small systems like pillar microcavities, phase correlations are present both below and above threshold, so that the spontaneous polarization remains the only direct measure of the order parameter.

IV. INTRODUCING EFFECTIVE MAGNETIC FIELDS

In experimental microcavity samples local strain effects and inhomogeneous photonic disorder produce effective magnetic fields acting on the pseudospins of polaritons. An effective magnetic field of these origins splits the linearly polarized states of the condensate. If the two condensates experience the same splitting frequency Ω, but the fields are oriented differently, the splitting Hamiltonian is given by

$$H_s = -\hbar\Omega[(\mathbf{c}_1\mathbf{S}_1)+(\mathbf{c}_2\mathbf{S}_2)], \tag{8}$$

where unit vectors $\mathbf{c}_{1,2}$ define orientations of the fields.

The introduction of the local fields modifies the distribution function of the condensate polarization as seen in Fig. 4. In this figure, points show the time-averaged values of the Stokes vector of the condensates for different realizations of the Langevin noise, corresponding to different excitation pulses. Here the splitting favors polarizations in the positive x and y directions for condensates 1 and 2, respectively. The splitting leads to a bias in the time-integrated values of the polarization and results in frustration of polarization locking. As expected this produces a reduction in the polarization correlation. The fields also introduce a bias in the phase relationship between the two condensates. $\phi_{2\sigma}-\phi_{1\sigma}$ becomes locked to either π or $-\pi$ depending on the pinning, which indicates that an antisymmetric solution for two Josephson-coupled condensates corresponds to the minimum energy.

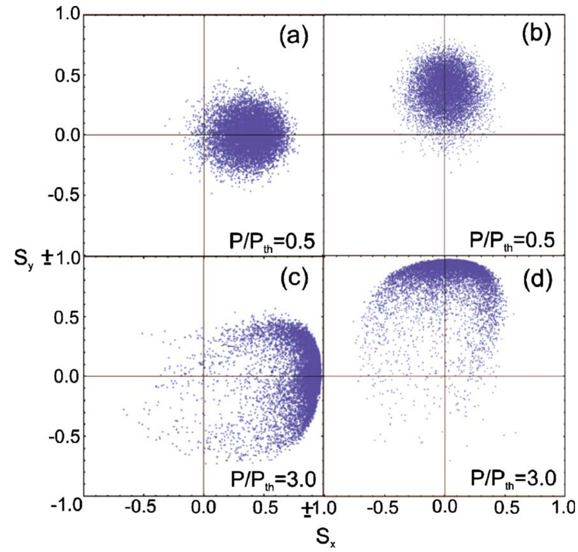

FIG. 4. (Color online) [(a) and (c)] Time-integrated polarization for condensate 1 below and well above threshold, respectively, where polarization is pinned to positive S_x. [(b) and (d)] Corresponding results for condensate 2 where polarization is pinned to positive S_y. The magnitude of applied effective fields is $\Omega = 1\ \text{ps}^{-1}$.

V. SLOW ENERGY RELAXATION OF CONDENSATE

We now consider the case where energy relaxation is slow and can be neglected over the lifetime of the condensate. To describe this regime we remove the final term of Eq. (2). Here the polarization correlation is not observed below or above threshold. This can be illustrated by considering the states 1 and 2 when the polaritons are condensed. If there is no tunneling between the states ($J=0$) then the order parameters are uncorrelated, $\langle \psi_1^\dagger \cdot \psi_2 \rangle = 0$. One can show that any linear combinations of ψ_1 and ψ_2, $\zeta_1 = u_{11}\psi_1 + u_{12}\psi_2$ and $\zeta_2 = u_{21}\psi_1 + u_{22}\psi_2$, are also uncorrelated provided the u coefficients form a unitary matrix.[17]

If Josephson tunneling is turned on, $J \neq 0$, the result is unchanged. In particular, symmetric and antisymmetric states $(\psi_1 \pm \psi_2)/\sqrt{2}$ are uncoupled and therefore uncorrelated. The unitary transformations of these states, in particular, ψ_1 and ψ_2 are also uncorrelated. Physically it means that any combination of the condensates wave functions can be spontaneously formed. The role of relaxation is to bring the system into a particular state that minimizes the Hamiltonian energy, e.g., into the antisymmetric $(\psi_1 - \psi_2)/\sqrt{2}$ state. No correlations are seen when the system randomly occupies symmetric and antisymmetric states, but they appear if the system relaxes into, e.g., an antisymmetric state. This theoretical argument is corroborated by numerical results.

VI. CONCLUSION

We have described theoretically the process of polarization locking over spatially separated polariton condensates by a Josephson coupling mechanism. The effect manifests itself as an increase in the two-emitter polarization correlator above threshold pumping power for a pulsed setup. A significant effect is seen at all times when the condensates are macroscopically occupied. Correlations are seen between the condensate phases showing the buildup of spatial coherence across the sample. We take into account the effect of differently oriented local effective fields acting on the condensates and see the frustration of correlations due to the polarization pinning. The presence of energy relaxation is shown as necessary for correlations to exist. These results show that the buildup of common vector polarizations can be considered as a criterion for appearance of a macroscopically coherent state in a system of localized polariton condensates. This criterion is as strict as the appearance of spatial coherence in a large-size system and becomes more rigorous than the spatial coherence criterion in the case of strongly localized condensates.

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